Calculus II — *Spring 2004*
MA114:004-006 — *Russell Brown*

1. Evaluate definite and indefinite integrals by substitution.
2. Find inverse functions graphically and algebraically.
3. Compute the derivative of the inverse of a function.
4. Compute integrals and derivatives involving logarithms.
5. Use definition to estimate $log x$ and establish properties.
6. Compute integrals and derivatives involving the exponential functions
7. Find solutions of constant coefficient differential equations when the solution is of the form e^{rx}.
8. Solve equations involving exponentials of arbitrary base.
9. Evaluate derivatives and integrals involving a^x.
10. Set up and solve problems related to exponential growth and decay, including problems about half-life.
11. Solve the differential equation $y' = ky$.
12. Use definitions of the inverse trigonometric functions, sin^{-1}, cos^{-1} and tan^{-1}.
13. Simplify expressions of the form $f(g^{-1}(x))$ where f and g are trigonometric functions.
14. Compute derivatives and integrals involving inverse trigonometric functions. One should know both the derivatives and understand how to use the material from section 6.1 to find these derivatives.
15. Recognize indeterminate forms. Especially, $0/0$, ∞/∞. Be able to reduce limits of the form $0 \cdot \infty$, and 1^∞ to evaluating a limit of a quotient.
16. State L'Hopital's rule for quotients.
17. Compute limits using L'Hopital's rule.

Sample problems

1. If
$$\int_2^4 x f(x^2)\, dx = 5,$$
can you find the value of
$$\int_4^{16} f(x)\, dx?$$

2. Find
$$\int_0^{\pi/2} \sin(x)\cos^3(x)\,dx$$

3. Let $f(x) = x^3 + 3x + 1$ and let $g(x)$ be the inverse function to f.
   (a) Show that f is one-to-one.
   (b) Find $g(5)$.
   (c) Find $g'(5)$.

4. Let $f(x) = \sin(x)$ for $\pi/2 \le x \le 3\pi/2$ and let $g(x)$ be the inverse of f.
   (a) Sketch the graph of $g(x)$.
   (b) Find $g'(x)$. Simplify your answer.

5. Compute the derivatives of the following functions.
$$\ln(\ln x)), \quad \frac{\ln x}{x}, \quad x\ln x - x.$$

6. Show that $\ln(2x) - \ln x$ is constant for x in $(0, \infty)$. You should be able to give two solutions.

7. Compute the integrals:
$$\int_0^2 xe^{x^2}\,dx, \quad \int \frac{e^x}{e^x+1}\,dx.$$

8. Find the forty-fifth derivative of $f(x) = e^{2x}$.

9. Solve the equation $200 = 3^x$. Express your answer exactly using the natural logarithm function and then give a decimal approximation to the answer.

10. Find the area between the curve $y = 2^x$ and the line $y = x + 1$.

11. Prove that if a function $y(x)$ satisfies $y' = 2xy$, then $y(x) = Ae^{x^2}$.

12. Suppose that a critter expects to reach an adult height of 2 meters. Let $h(t)$ be the height at time t years after birth and $d(t) = 2 - h(t)$. It is believed that the rate of change of d is proportional to d with constant of proportionality k. a) Write a differential equation for the quantity h. b) Is it sensible for the constant k to be positive or negative? Explain.

13. Suppose that the mass of a substance decays exponentially and that the mass decays from 100 grams to 25 grams in 31 days. a) Write a function that gives the mass of this substance after t days. (Assume that $t = 0$ corresponds to the time when the mass is 100 grams.) b) When will the mass be 33 grams?

14. If $\cos u = 5/13$ and u lies in the interval $[-\pi, 0]$, find the values of
$$\sin u, \quad \sec u, \quad \tan u, \quad \cos 2u, \quad \tan 2u.$$

15. Is it always true that $\sin(\sin^{-1} x) = x$? Is it always true that $\sin^{-1}(sinx) = x$?

16. Evaluate the integrals

$$\int_1^3 \frac{x}{x^2-4x+5}\,dx, \quad \int \frac{x+1}{\sqrt{4-x^2}}\,dx.$$

17. Suppose that the hypotenuse of a right tringle is fixed at 13 meters and one of the shorter sides, x, is increasing at a rate of 0.3 meters/second. Find the rate of change of the angle opposite x when x is 5 meters long.

18. State L'Hopital's rule for limits of the form

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}.$$

19. Compute the following limits:

$$\lim_{x\to 0} xe^{-x}, \quad \lim_{x\to 0} \frac{1-\cos x}{x^2}, \quad \lim_{x\to 0} (1+2x)^{1/x}.$$

20. Give an example of two functions f and g for which

$$\lim_{x\to 0} \frac{f(x)}{g(x)} \neq \lim_{x\to 0} \frac{f'(x)}{g'(x)}.$$