Calculus II | *Spring 2004*
MA114:004-006 | *Russell Brown*

The final will be cumulative with an emphasis on recent material. Every question from the first three parts of the course will be taken from a review sheet or from the old tests. I expect that about 40% of the final will cover new material and 60% will cover material from the first three exams.

The final exam will be from 10:30am-12:30pm in CP 320 on Wednesday, 5 May 2004.

In preparing the final, I will make use of the this review sheet, the three review sheets that I have prepared this semester and the three exams we have written. You are encouraged to review your solutions of these problems. Copies of (corrected) review sheets are available at www.math.uky.edu/∼rbrown/courses/ma114.s.04 and you may also find a link to a page of old exams at this site.

Topics.

1. Know items 1–17 on the integral table in the back cover of Stewart. Know is defined to mean 1. memorize the formula 2. be able to explain why the formula is true.
2. Know the trigonometric identities as discussed on the trigonometry handout for the first examination.
3. Definition of Taylor and McLaurin series.
4. Taylor's theorem with remainder and its use in estimating the error when approximating a series by a partial sum.
5. Know the McLaurin series for e^x, $\sin x$ and $\cos x$.
6. Sketch parametric curves.
7. Eliminate the parameter and find cartesian equations for a curve.
8. Find tangents to parametric curves.
9. Find arc length of parametric curves.

Some problems.

1. Evaluate the integral $\int_0^2 |t-1|\,dt$.
2. State Taylor's theorem with remainder.
3. Use Taylor's theorem with remainder to estimate $1/e$ with an error of less than 10^{-4}.
4. Use Taylor's formula for the remainder to find an upper bound for $|\cos 2 - P_5(2)|$ where P_5 is the fifth degree Taylor polynomial at 0 for $\cos x$.

5. Find the Taylor polynomial of degree 5 centered at 0 for the function $x/(1+x^2)$. Hint: Don't differentiate anything.

6. Find the Taylor polynomial of degree 15 for
$$\int_0^x \frac{1}{(1+t^4)}\,dt.$$

7. Write out the Taylor series for $f(x) = \sin x$ at $\pi/4$.

8. Estimate the integral $\int_0^1 \sin(x^2)\,dx$ to within 10^{-4}.

9. Find the tangent line to the curve $x(t) = t^2$, $y(t) = t^3$ which is parallel to the line $y = 2x$.

10. Find the tangent line to the curve $x(t) = t^2$, $y(t) = t^3$ which passes through $(1, 0)$. (Warning: I have not worked this problem.)

11. Set up an integral which gives the length of the ellipse $x(t) = 2\cos t$ and $y(t) = \sin t$. Use Simpson's rule with $n = 6$ to approximate the length.

    Can you evaluate the integral exactly?

12. Find the length of the curve $x(t) = t^2$ and $y(t) = t^3$ between $(1, 1)$ and $(4, 8)$.

13. Find the length of the curve $x(t) = \int_0^t \sqrt{\cos^2 s - s^2}\,ds$ and $y(t) = \cos t$ for $0 \le t \le 1/2$.

14. Find the length of the curve $y = \sqrt{1-x^2}$ for $0 \le x \le 1$.