Abstracts for the 59th Midwest Partial Differential equations seminar.

24–25 March 2007
University of Kentucky
Lexington, Kentucky

The order of growth of the resonance counting function in Euclidean scattering

Tanya Christiansen
University of Missouri, Columbia

Resonances are complex numbers which may be considered analogs of eigenvalues for a class of operators with continuous spectrum. We study the resonance counting function for perturbations of the Laplacian on odd-dimensional Euclidean space. This is the number of resonances with norm at most r, and we are particularly interested in its rate of growth as $r \to \infty$. While upper bounds on the growth rate are fairly well-understood, lower bounds have proven more elusive. We show that for several classes of perturbations, the set of operators with resonance counting function having maximal order of growth is generic. The proofs use techniques from one and several complex variables.

On Stefan and Hele-Shaw problems with a degenerate phase

Marianne Korten
Kansas State University

In this talk I will survey new results about parabolic free boundary problems with a degenerate phase connected to Stefan problems: The first one establishes the measure theoretical version of the Rankine-Hugoniot condition at "most" points of the free boundary in the one phase Stefan problem under minimal smoothness assumptions (joint work with Donatella Danielli). The second one establishes intrinsic energy estimates for the temperature in the two phase Stefan problem (which are more delicate to obtain as one is working with signed solutions). Immediate applications are explicit expressions for the free boundary measures, leading to an existence theorem for the two-phase Stefan problem when initial data are signed measures plus some other qualitative results about the free boundary (joint work with Chuck Moore). Next I will describe complete existence and uniqueness results for the Hele-Shaw problem with a degenerate phase, which hold in any dimension, even for a non-simply connected injection slot, and cover both Dirichlet and Neumann boundary data. Under standard assumptions we obtain regularity results for the free boundary (joint work with Ivan Blank and Chuck Moore).

Some Liouville theorems and gradient estimates

Yan Yan Li
Rutgers University

The classical Liouville theorem says that a positive entire harmonic function must be a constant. We give a fully nonlinear version of it. This extension enables us to establish local gradient estimates of solutions to general conformally invariant fully nonlinear elliptic equations of second order. This talk will start from a proof of the classical Liouville theorem using only the comparison principle and the invariance of harmonicity under Mobius transformations and scalar multiplications. We will then outline the proof of the comparison principle used in establishing the new Liouville theorem. Finally we outline the proof of the gradient estimates via the Liouville theorem.

A class of two-phase free boundary problems in the plane.

Arshak Petrosyan
Purdue University

We will talk about recent results in two-phase Alt-Phillips and obstacle-type free-boundary problems in two dimensions. In particular, we will show how to overcome the lack of nondegeneracy in the negative phase by a careful iteration of the sharp version of the Harnack inequality combined with topological arguments to establish the full regularity of the free boundary.

Inverse Problems in Geometric Scattering

Antônio Sá Barreto
Purdue University

One of the main questions in inverse scattering is to obtain information about the scatterer from the scattering matrix. We will describe some results on the question of determining certain Riemannian manifolds by the scattering matrix at all energies.

The mass-critical nonlinear Schrodinger equation.

Monica Visan
IAS

We discuss recent results on global well-posedness and scattering for the defocusing mass-critical nonlinear Schrodinger equation. This is joint work with Terence Tao and Xiaoyi Zhang.

Multi-dimensional Riemann problems for the compressible Euler system

Yuxi Zheng
Pennsylvania State University

We present results on solutions to the Euler system in two space dimensions with special data which include the Mach experiment and binary interactions of planar rarefaction waves.