

COMBINATORICS 2010: ADVANCES, TRENDS AND SPECULATIONS

(CATS 2010 WORKSHOP)

FRIDAY, MARCH 26, 2010, CB 106, UNIVERSITY OF KENTUCKY

9:00 am Invited address: Sara Billey
“A stratification of branched polymer space”

A mathematical model of polymers (from chemistry) has led to some very interesting questions in probability and algebraic geometry. We will introduce the basic concepts, describe some new results and give several open problems. This is joint work with Dave Anderson.

10:00 am Coffee and discussion

11:00 am Invited address: Isabella Novik
“Lower bound theorems for spheres, manifolds, and pseudomanifolds”

The celebrated lower bound theorem (due to Barnette, 1973 and Kalai, 1987) asserts that among all simplicial manifolds of a fixed dimension and with a fixed number of vertices, (the boundary complex) of a certain polytope, called a stacked polytope, simultaneously minimizes all the face numbers. In this talk we will discuss various strengthenings and generalizations of this theorem for the classes of (i) balanced simplicial spheres, (ii) general simplicial manifolds with and without boundary, and (iii) certain pseudomanifolds with isolated singularities. We will also briefly outline several commutative algebra techniques and new results that are needed for the proofs. Part (i) is joint with Michael Goff and Steve Klee; parts (ii) and (iii) are joint with Ed Swartz.

12:00 pm Lunch

2:00 pm Invited address: Patricia Hersh
“A combinatorial topological toolkit for stratified spaces”

There are quite a few stratified spaces, i.e., spaces made of pieces each homeomorphic to R^n for some n , of current interest in such areas as combinatorial representation theory, (real) Schubert calculus, algebraic statistics, and total positivity theory, the last of which are closely related to the theory of canonical bases and the theory of cluster algebras. A common feature of these spaces is that they may often be regarded as the image of a map

from a much simpler space of parameters. I will discuss some recently developed tools for determining topological structure of such spaces in a rather strong sense, namely homeomorphism type. This will include topological collapsing lemmas and a new criterion for determining whether a finite CW complex is regular with respect to a choice of characteristic maps. While the proofs are topological, using these tools can be essentially combinatorial, as I will illustrate on the main application to date.

3:00 pm Coffee and discussion

4:00 pm Invited address: Steph van Willigenburg
“Quasisymmetric refinements of Schur functions”

In this talk we introduce a basis for quasisymmetric functions, called quasisymmetric Schur (QS) functions, which partition Schur functions in a natural way. Furthermore, we show how these QS functions refine many Schur function properties including

- * Kostka numbers
- * Pieri rules
- * The Littlewood-Richardson rule.

Extending the definition of QS functions, we define skew QS functions, which likewise partition skew Schur functions. We observe how these skew functions arise in the study of both the noncommutative Schur functions of Rosas-Sagan, and the unrelated ones of Fomin-Greene.

This is joint work with Christine Bessenrodt, Jim Haglund, Sarah Mason, and especially Kurt Luoto, who will speak in the AMS Special Session on a noncommutative Littlewood-Richardson rule arising from skew QS functions and their relation to free Schur functions.

6:00 pm Workshop dinner: Rincon Mexicano

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