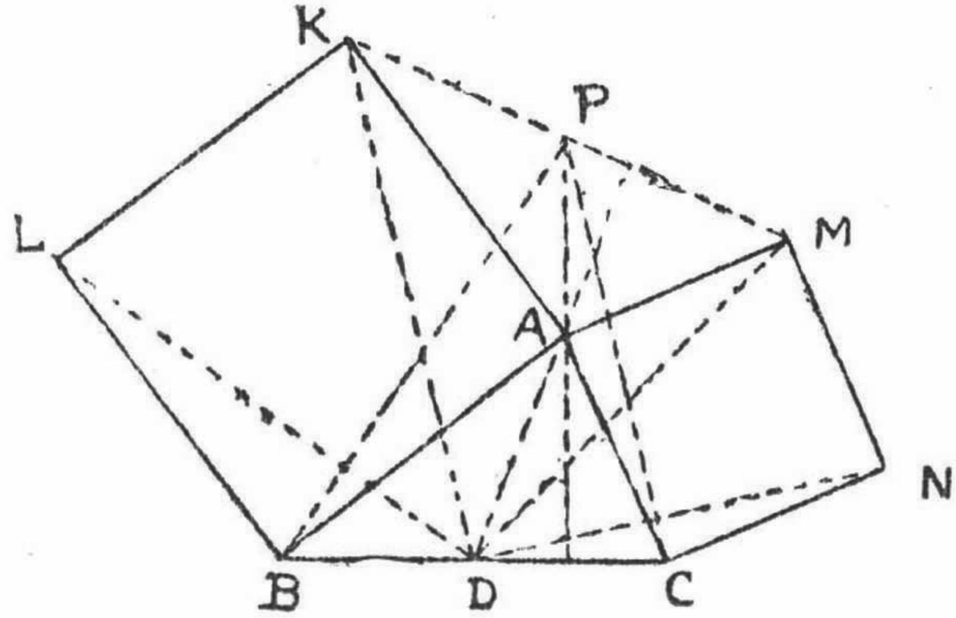
New Proofs of Old Theorems.

1. Apollonius' Theorem.

ABC is a triangle, D the mid. point of BC, ABLK, ACNM squares described externally on the sides AB, AC respectively, P the



mid. point of KM and PA, DA, DL, DK, DM, DN are joined as in the figure.

It is easily seen that PA is perpendicular to and equal to $\frac{1}{2}$ BC, and DA is perpendicular to and equal to $\frac{1}{2}$ KM.

Now, square $ABLK = 2\,\Delta\,LBD + 2\,\Delta\,KAD$

$$= 2\,\Delta\,BAP + 2\,\Delta\,KAD;$$

for, in the two triangles LBD, BAP, we have

$$LB = BA$$

$$BD = AP$$

and $$L\hat{B}D = B\hat{A}P$$

since LB, BD are perpendicular to BA, AP respectively and therefore

$$\Delta\,LBD \equiv \Delta\,BAP.$$

Similarly, square $ACNM = 2\,\Delta\,CAP + 2\,\Delta\,MAD.$

$$\therefore\ AB^2 + AC^2 = 2(\Delta\,BAP + \Delta\,CAP) + 2(\Delta\,KAD + \Delta\,MAD)$$

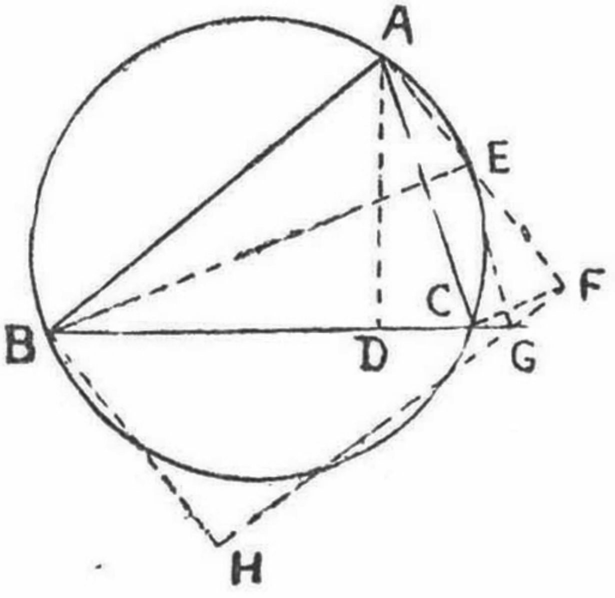
$$= AP\,.\,BC + AD\,.\,KM$$

$$= \tfrac{1}{2}\,BC^2 + 2\,AD^2$$

$$= 2\,BD^2 + 2\,AD^2.$$

2. Brahmagupta's Theorem.

ABC is a triangle inscribed in a circle, BE the diameter through B, and AD perpendicular to BC. AF is drawn perpendicular to BA and



on the same side of AB as the vertex C. AF is made equal to AC and the rectangle BAFH completed. BC produced meets FH at G and EG, CF are joined.

$$\text{Rect. } AB \cdot AC = \text{rect. } AH$$
$$= 2\,\triangle\, BAG$$
$$= \text{Rect. } BG \cdot AD.$$

From the properties of the cyclic quadrilateral,

$$\widehat{EBG} = \widehat{CAF}$$

and
$$\widehat{EGB} = \widehat{AFC},$$

so that the remaining angles of the two triangles EBG, ACF are equal,

viz.
$$\widehat{BEG} = \widehat{ACF}.$$

But
$$\widehat{AFC} = \widehat{ACF} \text{ since } AC = AF$$

$$\therefore \quad \widehat{EGB} = \widehat{BEG}$$

$$\therefore \quad BE = BG,$$

so that
$$\text{Rect. } AB \cdot AC = \text{Rect. } BE \cdot AD.$$

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