

# DEPARTMENT OF MATHEMATICS

Ma 110 First Exam September 26, 2008

**DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.**

**Instructions:** Be sure your name, section number, and student ID are filled in below. Cell phones must be OFF and put away before you open this exam. You may use calculators (including graphing calculators, but no laptops or cellphone calculators) for checking numerical calculations. However, you must show your work to receive credit.

Put your answers in the answer boxes provided, and show your work.

**If your answer is not in the box or if you have no work to support your answer, you will receive no credit.**

The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
Homework	10	
Total	100	

Please fill in the information below.

NAME: \_\_\_\_\_ Section: \_\_\_\_\_

Last four digits of Student ID: \_\_\_\_\_

1. Answer each of the following. Be careful to show your work.
- a) If  $X$  is a complex number and  $(3 + 2i)X + 5 + i = 0$  then:

$X =$
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- (b) Determine whether  $\frac{3+2i}{5+i} = \frac{5-i}{4+6i}$ .

**Explain your answer. Your explanation is what will be graded!**

Answer:
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2. Answer each of the following. Be careful to show your work.

Suppose  $f(x)$ ,  $g(x)$  and  $h(x)$  are polynomials such that:

(i)  $f(x)$  has degree 5

(ii)  $g(x)$  has degree 7 and

(iii)  $h(x)$  has degree 10.

(a) Then the degree of  $f(x)g(x)$  is

Answer:

(b) The degree of  $f(12g(x) - 11h(x))$  is

Answer:

(c) If  $h(x)$  is divided by  $f(x)$  using **long division** to write  $h(x) = f(x)q(x) + r(x)$  then, assuming it is not 0, the possible degrees for  $r(x)$  are

Answer:

and the degree of  $q(x)$  is

Answer:

3. Answer each of the following. Be careful to show your work.

(a) If  $f(x) = x^5 - 3x^4 - 18x^3 + 54x^2 + 81x - 243$  then **the coefficient of  $x^5$**  in

$(x + 5)f(x)$  is:

(b) If  $f(x) = x^5 - 3x^4 - 18x^3 + 54x^2 + 81x - 243$  and  
 $(x - a)f(x) = x^6 + x^5 - 30x^4 - 18x^3 + 297x^2 + 81x - 972$ , then

4. (a) Suppose  $f(x) = x^2 - 3x + 6$ . Determine the value of  $s$  such that  $f(u + s) = u^2 + C$  where  $C$  is a constant. In other words, in the expansion of  $f(u + s)$  the coefficient of  $u$  is zero.

Value of  $s$ :

- (b) With the value of  $s$  from part (a) the resulting value of  $C$  is

Value of  $C$ :

5. a) If  $(5p - y)^7$  is expanded and simplified, then the coefficient of  $p^2y^5$  will be

Answer:
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b) Calculate the degree and the lead coefficient of  $(x - 4)^{16} - (x + 2)^{16}$

degree:
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leading coefficient:
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6. Solve the system of equations:

$$\begin{cases} 4x + 10y = 9 \\ -3x - 4y = 1 \end{cases}$$

by Cramer's Rule.

Express each of  $x$  and  $y$  as the ratio of determinants of matrices and calculate each of the individual determinants.

**It is important to display the correct determinants before evaluating them.**

$$x = \frac{\det \begin{vmatrix} \square & \square \\ \square & \square \end{vmatrix}}{\det \begin{vmatrix} \square & \square \\ \square & \square \end{vmatrix}} \quad y = \frac{\det \begin{vmatrix} \square & \square \\ \square & \square \end{vmatrix}}{\det \begin{vmatrix} \square & \square \\ \square & \square \end{vmatrix}}$$

**Evaluated Answers:**

$x =$	$y =$
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7. A grocery shelf contains cans of red beans and cans of black beans. All cans of red beans have the same weight and all cans of black beans have the same weight. A shopper purchases 4 cans of red beans and 4 cans of black beans and her purchase weighs a total of 28 pounds. Another shopper purchases 2 cans of red beans and 3 cans of black beans and his purchase weighs a total of 19 pounds. Determine the weight (in pounds) of each can of red beans and of each can of black beans.

a) The weight of one can of **red beans (in pounds)** is:

Answer:

b) The weight of one can of **black beans (in pounds)** is:

Answer:



8. Fill in the missing entries in the following Aryabhata Algorithm table

$$\begin{pmatrix} \text{-Quotients} & \text{Answer 1} & \text{Answer 2} & \text{Integers} \\ \text{Begin} & 1 & 0 & 245 \\ -1 & 0 & 1 & 168 \\ \text{-----} & 1 & \text{-----} & \text{-----} \\ \text{-----} & -2 & \text{-----} & \text{-----} \\ \text{-----} & 11 & \text{-----} & \text{-----} \\ \text{End} & -24 & \text{-----} & \text{-----} \end{pmatrix}$$

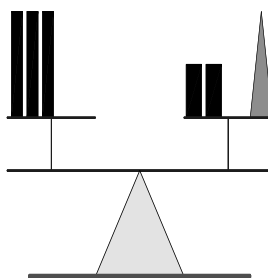
The greatest common divisor (GCD) of 245 and 168 is \_\_\_\_\_

From the table we can write GCD as a linear combination of 245 and 168 .

GCD = \_\_\_\_\_\*245 + \_\_\_\_\_\*168

The least common multiple (LCM) of 245 and 168 is \_\_\_\_\_

9.



A merchant has a balance, a large collection of 37 gram cylinders and another large collection of 29 gram cylinders. He also has an object which is supposed to weigh exactly 1 gram and he wants to check the weight. Study the following Aryabhata table and use it to find at least one way of checking the weight.

(	-Quotients	Answer 1	Answer 2	Integers	)
	Begin	1	0	37	
	-1	0	1	29	
	-3	1	-1	8	
	-1	-3	4	5	
	-1	4	-5	3	
	-1	-7	9	2	
	-2	11	-14	1	
	End	-29	37	0	

He checks the weight by putting the 1 gram object in the right pan as shown and balancing the pans by adding some of his cylinders in the pans as follows.

He places \_\_\_\_\_ cylinders of weight \_\_\_\_\_ grams in the left pan

and \_\_\_\_\_ cylinders of weight \_\_\_\_\_ grams in the right pan.

**Extra.** Can you find another solution?

## 1 Answer Key for ex1f08\_v1

1.  $\diamond$  (a)  $-\frac{17}{13} + \frac{7}{13}i$  (b) *false*
2.  $\diamond$  (a) 12 (b) 50 (c) [0, 1, 2, 3, 4] 5
3.  $\diamond$  (a) 2 (b) -4
4.  $\diamond$  (a)  $3/2$  (b)  $\frac{15}{4}$
5.  $\diamond$  (a) -525 (b) [15, -96]
6.  $\diamond$   $x = -46$   $14 - \frac{23}{7}$   $y = 31$   $14 \frac{31}{14}$
7.  $\diamond$  a) 2 pounds a) 5 pounds
8.  $\diamond$  GCD 7 COEFFS 11 -16
9.  $\diamond$  Left pan: 11 cylinders of weight 37 gms and Right pan: 14 cylinders of weight 29 gms.