DEPARTMENT OF MATHEMATICS

Ma162 FIRST EXAM Spring 2004
Solutions: February 16, 2004

1. Answer the following questions.

   (a) Find an equation of the straight line that connects $P(1, 2)$ and $Q(-2, -4)$. ANS:$y = 2\,x$

   (b) Where does the above line cross the x axis? Give both the coordinates of the point. ANS:$x = 0, y = 0$.

   (c) A line K is known to be parallel to the line $2x + 3y = 5$ and it is known that the point $P(-1, -1)$ is on K. What is an equation of K? ANS:$2\,x + 3\,y = -5$

   (d) A line L is known to be perpendicular to the line $-3x + 2y = 5$ and is known to cross the x axis at $x = -\frac{5}{2}$. What is an equation of L? ANS:$2\,x + 3\,y = -5$

2. A factory has three branches each manufacturing product lines called A, B, C. The production rate per hour for each branch is given in the following table.

	A	B	C
Branch 1	3	4	1
Branch 2	2	0	5
Branch 3	1	3	4

Create appropriate equations to determine the number of hours each branch must run in order to meet the following conditions.

Let x_1, x_2, x_3 denote the number of hours the branches 1,2,3 are run respectively.

The factory needs a total of 116 units of product A, 170 units of product B and 250 units of product C.

Product A equation: $3x_1 + 2x_2 + x_3 = 116$.

Product B equation: $4x_1 + (0)x_2 + 3x_3 = 170$.

Product C equation: $1x_1 + (5)x_2 + 4x_3 = 250$. Be sure to identify the variables used.

Set up the appropriate augmented matrix that you would have to solve for finding the solution of the equations. **Do not solve!**

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 116 \\ 4 & 0 & 3 & 170 \\ 1 & 5 & 4 & 250 \end{array}\right]$$

3. A publisher sells a certain book for \$ 7.00, and he needs to spend 50 cents per book for handling the sales. He also has a fixed production cost of \$ 24000 per month and needs \$ 4.00 per book for printing costs.

   Answer the following questions:

   (a) Find the monthly cost function. **Be sure to describe the meaning of the variable that you use.**

   Let x be the number of books published.

   $C(x) = 4.5x + 24000$.

(b) Find the monthly revenue function.
$R(x) = 7x$.

(c) Find the monthly profit function.
$P(x) = R(x) - C(x) = 2.5x - 24000$.

(d) What is the net profit (loss if negative) if he print and sells 1000 books?
Loss: \$21500.

(e) How many books must be printed and sold to break even? **(Round up the answer to the next integer.)**
Ans: 9600.

4. Precisely list **the three allowable elementary row operations** on an augmented matrix of a system of equations. Be sure to use complete precise sentences and describe the operations in words, not just symbols.

(a) **Row-swap**. You can interchange any **two** rows of the matrix.

(b) **Scalar multiple** You can multiply any row by a **non zero** constant (called scalar).

(c) **Add-row** You can add any (scalar) multiple of a row to any **other** row.

Carry out the indicated elementary row operations on the given matrix M **in the specified order.**

$$M = \left[\begin{array}{rrr|r} 0 & 10 & 2 & 3 \\ -2 & 2 & 0 & 4 \\ 2 & 3 & 1 & 5 \end{array}\right]$$

(a) Swap the first and the third row. ($R_1 \leftrightarrow R_3$.)

$$\begin{bmatrix} 2 & 3 & 1 & 5 \\ -2 & 2 & 0 & 4 \\ 0 & 10 & 2 & 3 \end{bmatrix}$$

(b) Add the first row to the second ($R_2 + R_1$.)

$$\begin{bmatrix} 2 & 3 & 1 & 5 \\ 0 & 5 & 1 & 9 \\ 0 & 10 & 2 & 3 \end{bmatrix}$$

(c) Subtract two times the second row from the third. ($R_3 - 2R_2$.)

$$\begin{bmatrix} 2 & 3 & 1 & 5 \\ 0 & 5 & 1 & 9 \\ 0 & 0 & 0 & -15 \end{bmatrix}$$

(d) Since M represented a system of three equations in three variables, what more work is needed for finishing the solution process? **Just discuss what might be needed, don't carry out any further operations.**

The last equation reduces to $0 = -15$ and hence the system has no solution! The work is finished.

Additional comments: In case the last equation would be valid with nonzero entris on both sides, we would see three pivots in REF form. The number of variables is also three, so there would be a unique solution to be computed by back substitution, or by continuing to RREF. It is guaranteed that in RREF, the left hand side part of the matrix would be I_3.

5. The following matrix in in REF. Convert it to the row reduced form (RREF). **You must indicate the row operations used and show the steps.**

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 5 & 1 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 2 & -4 \end{array}\right]$$

The operations are: $1/2R_3, R_2 - 5R_3, R_1 - 5R_3$. (Intermediate steps are left for the reader!) The result is:

$$\begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 14 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

The following matrix is already row reduced (RREF). Write out a solution in parametric form.

$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & 3 & -1 \\ 0 & 0 & 1 & -2 & 5 \end{array}\right]$$

Let the variables be called x, y, z, w. The pivot variables are x, z. Solve the equations for these, to get:

$$x = -1 - 4y - 3w \ , \ z = 5 + 2w \quad y, w \text{ arbitrary.}$$

In a more traditional way, this is also written by picking two parameter names s, t:

$$x = -1 - 4s - 3t \ , \ y = s \ , \ z = 5 + 2t \ , \ w = t \text{ where } s, t \in \Re.$$

6. (a) Find a value of k which makes the given system have infinitely many solutions or be inconsistent. *You must show justification; just a yes or no will receive no credit!*

$$3x - 2y = 5 \ , \ kx + 4y = 7.$$

The system does not have a unique solution exactly when its determinant $(3)(4) - (-2)(k)$ is zero. **I am using the Cramer's Rule. Look it up!** This is $12 + 2k = 0$ or $k = -6$.

For this value of k, we get $3x - 2y = 5, -6x + 4y = 7$, which are two parallel lines, so the system is inconsistent!

(b) Decide whether or not $(-1,\ (3-3t),\ t)$ represents a parametric solution to the given system of equations. *You must show justification; just a yes or no will receive no credit!*

$$5x + 2y + 6z = 1, -2x + y + 3z = 5.$$

The substitution of the given parametrization gives identities $1 = 1$, $5 = 5$ for the given equations. Hence it is a parametrization (parametric solution).

7. Consider the following matrices.

$$A = \begin{bmatrix} 0 & 10 & 2 \\ -2 & 2 & 0 \\ 2 & 3 & 1 \end{bmatrix},\ X = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix},\ B = \begin{bmatrix} 16 \\ 2 \\ 10 \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

(a) Is $3A + 2B$ defined? Explain your answer. If it is defined, calculate it.

Undefined - different sizes.

(b) Is $3A + 2C$ defined? Explain your answer. If it is defined, calculate it.

$$\begin{bmatrix} 0 & 32 & 6 \\ -10 & 10 & 0 \\ 10 & 9 & 5 \end{bmatrix}$$

(c) Is AX defined? Explain your answer. If it is defined, calculate it and decide if $AX = B$.

$$AX = \begin{bmatrix} 16 \\ -2 \\ 10 \end{bmatrix}$$

This is different from B.